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Preschool Acuity of the Approximate Number System Correlates with School Math Ability

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Abstract

Previous research shows a correlation between individual differences in people's school math abilities and the accuracy with which they rapidly and nonverbally approximate how many items are in a scene. This finding is surprising because the Approximate Number System (ANS) underlying numerical estimation is shared with infants and non-human animals who never acquire formal mathematics. However, it remains unclear whether the link between individual differences in math ability and the ANS depends on formal mathematics instruction. Earlier studies demonstrating this link tested participants only after they had received many years of mathematics education, or assessed participants' ANS acuity using tasks that required additional symbolic or arithmetic processing similar to that required in standardized math tests. To ask whether the ANS and math ability are linked early in life, we measured the ANS acuity of 200 3- to 5-year-old children using a task that did not also require symbol use or arithmetic calculation. We also measured children's math ability and vocabulary size prior to the onset of formal math instruction. We found that children's ANS acuity correlated with their math ability, even when age and verbal skills were controlled for. These findings provide evidence for a relationship between the primitive sense of number and math ability starting early in life.

Mathematical competence, from managing a budget to calculating a restaurant tip, is essential to everyday activity in most modern cultures. Indeed, previous research suggests that math ability is an important factor in determining career success, income, and psychological well-being (Paglin & Rufolo, 1990; Parsons & Bynner, 2005; Rivera-Batiz, 1992; Rose & Betts, 2004). Yet, wide variety exists in the level of mathematical competence that people achieve, even starting early in development. Investigations of school math ability, hereafter referred to as math ability, in kindergarteners and elementary school children find stable individual differences in performance on tasks relevant to school success (e.g., verbal counting, simple arithmetic, ordinal comparison of numerals, and story problems) (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Olah, & Locuniak, 2006; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Mazzocco & Thompson, 2005). These studies reveal individual differences in mathematical ability already present from the earliest years of formal education, and highlight the importance of investigating their sources. Moreover, identifying predictors of weak math abilities might allow for early detection of future math difficulties and a hastening of intervention.

What factors lead to early mathematical competence? In addition to social dimensions such as income level (Griffin, Case, & Siegler, 1994; Jordan et al., 2009), amount of number-relevant teacher input (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006), and home learning environment (Melhuish et al., 2008), cognitive capacities contribute significantly. General cognitive abilities such as short-term and working memory have been

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suggested to play an important role in mathematical abilities (Geary, 2004; Mabott & Bisanz, 2008; McLean & Hitch, 1999; Passolunghi & Siegel, 2001; Wilson & Swanson, 2001). In addition, recent attention has been given to the possibility of individual differences in an unlearned, number-specific competence used by children and adults.

Educators and researchers often refer to a “number sense” that includes a variety of competences, including the ability to subitize and count, to discriminate quantities, to discern number patterns, to rule out unreasonable results of arithmetic operations, and to move flexibly between different numerical formats (Berch, 2005; Gersten, Jordan, & Flojo, 2005; Jordan et al., 2007; Kalchman, Moss, & Case, 2001). This number sense supports math achievement and is a focus of many United States math curricula (NCTM, 2000; NMAP, 2008). One central component of the number sense is the Approximate Number System (ANS). A focus of research in cognitive psychology and neuroscience, the ANS has been shown to support a primitive sense of number in infants, children, and adults (for reviews see Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Libertus & Brannon, 2009). It is present at birth (Izard, Sann, Spelke, & Streri, 2009) and has been documented in many non-human animal species (for review see Brannon, Jordan, & Jones, 2010), supporting the notion that the ANS is independent from language and other acquired number symbols. In humans, the ANS is active across the entire lifespan, from infancy to old age (Dehaene, 1997; Halberda et al., 2011). Finally, a wealth of brain imaging studies has identified the intraparietal sulcus as the neural locus of the ANS (for review see Nieder & Dehaene, 2009).

The ANS has been shown to produce imperfect “noisy” estimates of numbers of items from input across all sensory modalities (e.g., beeps, visually or tactilely presented objects, taps of a finger). These numerical estimates support quantitative computations such as “greater-than, less-than,” addition, subtraction, multiplication, and division (Barth, Kanwisher, & Spelke, 2003; Barth et al., 2006; Barth, La Mont, Lipton, & Spelke, 2005; Eger, Sterzer, Russ, Giraud, & Kleinschmidt, 2003; McCrink & Spelke, 2010; McCrink & Wynn, 2004). The inherent noisiness of the ANS means that the accuracy of observers’ numerical estimates, and hence their performance at comparing or computing over ANS representations, accords with Weber’s Law, with larger numerical estimates being increasingly imprecise. As a result, the discriminability of any two ANS representations is a function of the ratio between them (e.g., 5 is as discriminable from 10 as 10 is discriminable from 20). Importantly, the amount of variability associated with representing a particular number (e.g., with approximating how many items are present when flashed an array containing exactly 10 dots) is not fixed over development. Infants and young children have much noisier ANS representations than adults, with the acuity of ANS representations sharpening throughout childhood, eventually supporting adult discriminations of about 9:10 (Halberda & Feigenson, 2008; Libertus & Brannon, 2010; Lipton & Spelke, 2003; Piazza et al., 2010; Xu & Spelke, 2000).

It remains unknown exactly when these noisy ANS representations integrate with more formal math abilities, and what role they may play. One provocative hypothesis is that the ANS is instrumental for acquiring symbolic numerical skills such as counting and arithmetic (Condry & Spelke, 2008; Dehaene, 1997; Dehaene, Dehaene-Lambertz, & Cohen, 1998; Gallistel & Gelman, 2000; Gilmore, McCarthy, & Spelke, 2007) (but see Butterworth, 2010; Carey, 2000). Another possibility is that the ANS is not required for early math understanding and only later is integrated with symbolic number representations (Le Corre & Carey, 2007).

By adolescence, the ANS appears to play a role in school mathematics performance. Halberda, Mazzocco, and Feigenson (2008) tested 14-year-old adolescents on a non-symbolic number comparison task in which participants saw rapidly flashed arrays of spatially intermixed blue and yellow dots, and pressed a key to indicate whether there were more blue or more yellow dots. As predicted by Weber's Law, adolescents' accuracy was modulated by the ratio between the numerical values -- the closer the two numerical values relative to each other, the lower the group accuracy. The authors then used psychophysical modeling to estimate each individual participant's Weber fraction (w, i.e., the amount of noise in each participant's ANS representations). ANS acuity measured by this simple task at age 14 years was found to significantly correlate with individual math ability all the way back to kindergarten, as measured by standardized math tasks administered throughout participants' schooling. Furthermore, this relationship remained robust even when controlling for non-numerical factors such as general IQ, spatial abilities, and working memory. Thus, it appears that individual differences in the ANS are linked to individual differences in performance on school math tasks, at least in adolescents.

This link also appears to modulate performance in participants who struggle with math. Piazza and colleagues (2010) and Mazzocco and colleagues (in press) demonstrated that children with dyscalculia have significantly worse ANS acuity than age-matched peers without dyscalulia. This suggests that less accurate ANS representations may be related to difficulty in school mathematics for children from the lowest end of math achievement.

Several investigations of children's ability to perform number line estimations suggest that the nature of ANS representations may be linked to math achievement even earlier in life. Booth and Siegler (2006, 2008) found that the spatial representation of the mental number line affects children's mathematical performance from as early as 5 years of age. In these studies, children were given Arabic numerals to place in their approximate spatial position on a schematic number line with only its anchor points (smallest and largest numbers) marked. The more linear (as opposed to logarithmic) children's representations were, the better their math ability. Hence the spatial organization of number representations may affect math abilities (Siegler & Ramani, 2009).

Other recent studies found evidence consistent with the idea that the ANS might affect ordinal decisions over numerical symbols, and thereby impact math ability. These studies tested children between 6 and 10 years of age on their rapid judgments of symbolic quantity (e.g., indicating whether the Arabic numeral 7 or 9 is numerically greater), and found that this ability was related to math abilities on calculation tests (De Smedt, Verschaffel, & Ghesquiere, 2009; Durand, Hulme, Larkin, & Snowling, 2005; Holloway & Ansari, 2008; Rousselle & Noel, 2007).

However, these number line and ordinal judgment tasks required children to process numerical symbols (e.g., Arabic numerals). As such, these studies do not yet answer the question of whether individual differences in ANS acuity per se, as opposed to individual differences in the processing of number symbols or integrating ANS representations with number symbols, correlate with formal math ability in early childhood, prior to the large amounts of formal math instruction that children receive in primary school.

Another recent piece of evidence for a link between the ANS and math ability comes from a study by Gilmore, McCarthy, and Spelke (2010). The authors tested 5-year-old children on standardized math tests as well as a non-symbolic addition task, in which children saw one quantity of blue dots added to another quantity of blue dots, and then had to report whether their approximate sum was more or less numerous than a comparison quantity of red dots. Children's accuracy on the non-symbolic addition task was positively correlated with their math ability scores, suggesting that individual differences in performance on non-symbolic arithmetic are related to math ability.

These previous studies are exciting and suggestive of a relationship between ANS and math ability starting from early in life. However, to date no study has focused on ANS acuity in young children in isolation of competence in other math-relevant abilities (e.g., representing a physical number line, processing Arabic digits, performing addition and subtraction). In the present investigation we aimed to test the relationship between ANS acuity and early math ability in young children who have received only minimal formal mathematics instruction. To this end, we tested 200 3- to 5-year-old children in a simple non-symbolic number comparison task that was used previously to demonstrate a relationship between ANS acuity and math achievement in 14-year-old children (Halberda et al., 2008). We also tested children on a standardized test of math ability (Test of Early Mathematics Ability, TEMA-3; Ginsburg & Baroody, 2003) and an assessment of verbal ability (Developmental Vocabulary Assessment for Parents, DVAP; Libertus, Stevenson, Odic, Feigenson, & Halberda, in preparation). Of primary interest was whether, even in young children with little or no formal mathematical instruction, individual differences in ANS acuity would correlate with math ability, with non-mathematical abilities controlled for.

Method

Participants

Two hundred children with a mean age of 4 years, 2 months (SD = 8.7 months; range = 33 months to 73 months; 97 females) participated. Twenty-six children were excluded from the final sample because of failure to complete the experiment (n = 16), external interference (n = 1), experimenter error (n = 2), insufficient understanding of English to follow task instructions (n = 2), incorrect age (n = 2), and atypical development (autism: n = 2, parent-reported cognitive deficits: n = 1) resulting in a sample of 174 children whose data were included in the final analyses. One-hundred-and-twelve of the children were tested in the laboratory, and the other eighty-eight were tested in a quiet room at their local preschools. Preliminary data analysis found no differences between these groups and so they were combined throughout.

To gain access to the preschools, letters were sent to the school directors requesting permission to conduct the experiment at the school. Once approval was obtained, information packages were distributed to parents describing the study and its purpose. Children who participated in the laboratory were recruited from commercially available lists and through flyers. Parents of all children tested provided informed written consent prior to their child’s participation in the experiment. Children, whether tested in the laboratory or at school, received a small gift (e.g., pencil, stickers) to thank them for their participation.

Materials

ANS acuity task—To measure the acuity of children’s Approximate Number System (ANS), we administered a non-symbolic numerical comparison task similar to the one previously employed by Halberda, et al. (2008). Children were presented with arrays of spatially separated blue and yellow dots on a 13-inch Apple MacBook laptop screen, and had to indicate whether more of the dots were blue or more of the dots were yellow (see Figure 1). The experimenter initiated each trial when the child appeared to be attentive. Each dot array was presented for 2000 ms followed by a blank screen until the child gave a verbal response (e.g., “yellow”). The experimenter immediately pressed the corresponding key on the keyboard (e.g., “y” for “yellow”), which recorded the answer and the response time from image onset until the key press. Two different sounds provided feedback about the correctness of the answer throughout the experiment. A high-pitched beep indicated a correct answer; a low-pitched beep indicated an incorrect answer. Children were familiarized to these sounds on six practice trials during which the experimenter provided additional feedback to ensure that children understood the task and were motivated to participate. Following these practice trials, a total of 60 test trials were presented.

The number of dots in each collection (blue and yellow) ranged from 4 to 15. Test trials were randomly drawn from one of four numerical ratio bins: 1:2, 2:3, 3:4, 6:7 (with the absolute number of dots on each trial varying, such that a trial with e.g., 5 yellow versus 10 blue dots would go into the 1:2 ratio bin). On half of the trials the yellow dots were more numerous; on the other half the blue dots were more numerous. On half of the trials the two colors were equated for individual dot size (i.e., the average size of the dots in each collection was equal). On the other half of the trials, the cumulative surface area of the blue dots and the yellow dots was equated. The default radius of the dots was 60 pixels and the maximum variability in size between the dots was +/−35%. The minimum distance between dots was 85 pixels from edge to edge.

Mathematical ability—To measure children's math ability we administered Form A of the Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2003). This test measures numbering skills (e.g., verbally counting the number of objects on a page), number-comparison facility (e.g., determining which of two spoken number words is larger), numeral literacy (e.g., reading Arabic numerals), mastery of number facts (e.g., retrieving multiplication facts), calculation skills (e.g., solving written addition and subtraction problems), and number concepts (e.g., answering how many tens are in one hundred). The TEMA-3 has been normed for children between the ages of 3 years 0 months and 8 years 11 months, and performance correlates highly with performance on other math achievement tests such as the math subtests of the Diagnostic Achievement Battery-Third Edition (Newcomber, 2001) and the Woodcock-Johnson III Tests of Achievement (Woodcock, McGrew, & Mather, 2001).

Verbal ability—We measured children's expressive vocabulary and controlled for this factor in our correlations. Parents of all participants completed the Developmental Vocabulary Assessment for Parents (DVAP; Libertus et al., in preparation). The DVAP consists of a list of the first 212 words from Form A of the Peabody Picture Vocabulary Test (PPVT-4; Dunn & Dunn, 2007). Parents were asked to indicate each listed word that they had ever heard their child say. An independent study found that 3- to 5-year-old children's DVAP scores correlate highly with their performance on the PPVT-4 (Libertus et al., in preparation), an experimenter-administered test which has been independently normed to measure the receptive language skills of children and adults starting at age 2.5 years. At present, the analysis of the relationship between DVAP scores and PPVT-4 performance includes usable data from 85 3–5-year-old children. The results show a significant relationship between DVAP and PPVT-4 scores ($R^2 = 0.68$, $p < 0.001$). We therefore used the DVAP rather than the PPVT-4 in the present study to shorten overall testing time.

Procedure

Two experienced testers conducted all testing sessions. Preliminary analyses found no differences as a function of experimenter and so all testing sessions were combined for subsequent analysis. Children completed the Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2003) and the ANS acuity task (i.e., the dots comparison task) during a single 30–45-minute testing session, with the TEMA-3 always administered first in order to reduce between-subject noise variability due to test order. Parents of children who were tested in the laboratory completed the Developmental Vocabulary Assessment for Parents (DVAP; Libertus et al., in preparation) before or during children's testing sessions. Parents of children who participated in preschools were given the DVAP form and asked to return it by email, fax, or regular mail.

Results

ANS Acuity Task

Children's performance on the Approximate Number System (ANS) acuity task was analyzed in terms of accuracy (percent correct) and response time (RT). Prior to analysis all trials in which RT was two or more standard deviations above or below a participant's mean RT were discarded as outliers (average = 4%, SD = 2.6%, range = 0% – 23.3%). Preliminary analyses found no effects of gender or perceptual variables (dot size and cumulative area of dots) on children's accuracy and RT when controlling for age (all Fs(1,171) < 2.54, *p*s > 0.11). Therefore, for all further analyses we collapsed across the trials in which individual dot size was equated and the trials in which cumulative area was equated, and did not include gender as a factor.

On average, children responded correctly on 65.10% of the trials (SD = 15.15%) and their average RT was 3205.13 ms (SD = 988.93 ms)[1]. As predicted by Weber's Law, children's accuracy decreased as the numerical ratio increased (i.e., a ratio of 1 occurs when the number of blue and yellow dots are equal; see Figure 1). We used a psychophysical model to estimate each participant's Weber fraction w (i.e., the amount of noise in each participant's underlying ANS representations; see e.g. Halberda & Feigenson, 2008; Halberda et al., 2008; Pica, Lemer, Izard, & Dehaene, 2004 for details). In this model, the numerosities of the two collections are modeled as Gaussian random variables with means n_1 and n_2 and standard deviations equal to w multiplied by the respective mean. Subtracting the Gaussian for the numerically smaller set from the numerically larger set yields a new Gaussian with a mean of $n_1 - n_2$ and a standard deviation of $w\sqrt{n_1^2 + n_2^2}$. Accuracy is modeled as 1 minus the error rate where the error rate is defined as the area under the tail of the resulting Gaussian: $\frac{1}{2} erfc\left(\frac{n_1 - n_2}{\sqrt{2}w\sqrt{n_1^2 + n_2^2}}\right)$. Using this method, the noise in each participant's mental ANS representations was estimated by the single free parameter w fit using the Levenberg-Marquardt algorithm for nonlinear least-squares fit on the average accuracy for each ratio bin. The model did not settle on a reliable fit for 52 out of 174 participants because of high variability in their responses[2]. We performed analyses on w just for participants for whom the model yielded a reliable fit (n = 122), but analyzed the accuracy for all useable participants (n = 174). As can be seen in Figure 1, the psychophysical model was able to capture the group performance on the ANS acuity task extremely well ($R^2 = 0.97$). The average w for our sample was 0.64 (SD = 0.49), which agrees well with other published estimates for this age range (Halberda & Feigenson, 2008; Piazza et al., 2010).

Mathematical Ability and Vocabulary Size

Children's average standardized score on the Test of Early Mathematics Ability (TEMA-3) was 107.43 (SD = 15.00). On the Developmental Vocabulary Assessment for Parents (DVAP), parents reported that their children used an average of 93.08 words (SD = 27.81). We were unable to obtain DVAP scores from 14 children due to parents' failure to return the forms, and therefore these participants were excluded from analyses involving DVAP scores.

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[1] We note that the experimenter (who pressed a key to input children's responses) was able to see the stimuli. This could have resulted in subtle RT effects based on the experimenter's own processing of the stimuli. However, all RT analyses reported here focus on children's average RT across correct and incorrect responses and across ratios. Using only RTs from correct trials (in which experimenter bias should be minimized) yields the same pattern of results as reported above.

[2] Inclusion of a random guessing parameter in the model as previously employed by Pica et al. (2004) and Halberda and Feigenson (2008) yielded the same pattern of results in all analyses.

Relationship Between ANS Acuity and Math Ability

More precise ANS representations have been shown to lead to faster, more accurate numerical judgments (Halberda et al., 2011). For this reason, individual differences in ANS acuity can be assessed by differences in accuracy (percent correct) on the ANS acuity task (i.e., dots task), in the Weber fraction (*w*) that is fit to each participant's accuracy as a function of numerical ratio, and in average response time (RT) across all trials of the ANS acuity task.

To ask whether ANS acuity significantly correlates with math ability in preschoolers we first correlated each of the three estimates of ANS acuity (i.e., accuracy, *w*, RT) with mathematics ability as measured by TEMA-3 scores. As can be seen in Figure 2, each of these estimates significantly correlated with math ability (accuracy: $R^2 = 0.18$, $p < 0.001$; *w*: $R^2 = 0.07$, $p < 0.01$; RT: $R^2 = 0.08$, $p < 0.001$). This means that faster RT and greater accuracy on the ANS acuity task are associated with higher math ability. These results extend the previously demonstrated relationship between ANS acuity and math ability found in 14-year-olds (Halberda et al., 2008), and demonstrates that this relationship holds during the preschool years, prior to children receiving formal school instruction in mathematics.

To ensure that the relationship between ANS acuity and math ability was specific to mathematics, we ran two additional regression analyses controlling for age of the child at the time of testing and vocabulary size as measured by DVAP scores. These analyses included RT in the ANS task as an additional factor along with accuracy (model 1) and with *w* (model 2), thereby allowing us to control for any speed-accuracy tradeoffs[3]. All independent factors in each model were controlled for age and then correlated with math ability (TEMA-3 scores) focusing on the partial correlations. Partial correlations represent the proportion of variance in math ability scores explained by the listed variable when controlling for the remaining variables. As can be seen in Table 1, the two models captured a significant amount of variance in children's math ability. Most importantly, the two estimates of ANS acuity in each model (i.e., accuracy, *w,* and RT) contributed uniquely to the relationship with math ability when controlling for age and vocabulary size. This reveals that, accuracy, *w,* and RT each carry information about ANS acuity and that the relationship between math ability and ANS acuity remains significant when controlled for age and vocabulary size.

Discussion

Our findings reveal that the acuity of preschoolers' Approximate Number System (ANS) correlates with their school math ability. The robustness of this relationship was established in two ways. First, we found that three different estimates of ANS acuity (accuracy, *w,* and RT) correlate with math ability as measured by the Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2003) in preschoolers. Second, these correlations between ANS acuity and math ability remain significant when controlling for age, vocabulary size, and speed-accuracy trade-offs in the ANS acuity task. These findings thus provide strong evidence for a link between ANS acuity and math ability early in life.

Previous studies testing older children left open the possibility that differences in instructional experience mediated both increases in symbolic math abilities and in ANS acuity (Halberda et al., 2008; Piazza et al., 2010). However unlike previous studies, the present results show that the link between ANS acuity and math ability is already present before the beginning of formal math instruction.

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[3] The reason to run two separate models is that *w* is derived from accuracy and including these factors together in a single model would violate independence.

Our findings also demonstrate that the relationship between ANS acuity and math ability holds even when these are measured using highly non-overlapping tasks. Previous reports of a relationship between primitive numerical abilities and math often used tasks that overlapped in the need to process Arabic digits, or to perform explicit addition and subtraction (Booth & Siegler, 2006, 2008; De Smedt et al., 2009; Durand et al., 2005; Gilmore et al., 2010; Holloway & Ansari, 2008). Therefore, our results more directly bolster the conclusion that individual differences in the noisiness of people's ANS representations *per se* are linked to individual differences in their math ability.

However, our study leaves open the root cause of the link between ANS acuity and math ability. One possibility is that the ANS is foundational for acquiring symbolic numerical abilities (e.g. Dehaene, 1997). It is conceivable that while learning the meaning of number words, children need to map these to corresponding representations in the ANS (Dehaene, 1997; Gallistel & Gelman, 1992; Mundy & Gilmore, 2009) (but see Le Corre & Carey, 2007). On such a view, children's ANS acuity may have important consequences for the ease with which children acquire the counting sequence and other subsequent symbolic numerical skills, and thereby could affect the robustness of symbolic number representations. Another possibility is that noisier ANS representations may cause difficulties in performing and evaluating arithmetic operations (Gilmore et al., 2007, 2010). Yet a third possibility is that less accurate representations in the ANS may lead to decreased engagement in number-related activities, which might cause an increase in math anxiety and a subsequent decrease in math performance (Maloney, Ansari, & Fugelsang, 2011; Maloney, Risko, Ansari, & Fugelsang, 2010). Although here we show a connection between ANS acuity and symbolic numerical abilities in preschoolers, our study cannot yet elucidate the origin of this link.

It is also possible that the size of the relationship between math ability and ANS acuity observed in this study is smaller than what has been reported previously. For example, Halberda and colleagues (2008) found that w explained between 14 and 20 percent of the variance in math ability on a standardized math test when controlling for other cognitive abilities such as intelligence, task demands, memory, and language. In the present study, while accuracy, w, and RT explained between 7 and 18 percent of the variance before controlling for age and vocabulary size, entering these factors into a single model resulted in smaller effect sizes for w (6 percent explained) and accuracy (13 percent explained) while RT continued to predict between 5 and 8 percent of the variance in math ability. An important consideration is that both RT and estimates of accuracy (percent correct and w) appear to carry information about ANS acuity in the current dataset, and RT was not previously analyzed in adolescents (Halberda et al., 2008). Considering the predictive power of the conjunction of these factors for the current dataset suggests that the effect size for the relationship between ANS acuity and math ability in the present sample is no less powerful than what was previously demonstrated for adolescents (Halberda et al., 2008). Future studies that rely on parallel methods across ages will be needed to determine the relative changes in the effect size of this relationship throughout the lifespan; the current results highlight that considering both w and RT will be important for such analyses.

A few further limitations of our study raise the need for future research. First, as noted above, it is unclear whether acuity of the ANS *predicts* math abilities and what role the ANS plays in acquiring symbolic numerical abilities. Our cross-sectional design does not shed light on this important question, and future longitudinal studies will be needed to fully address this issue. Second, we did not directly control for overall intelligence, information-processing speed, working memory, or other cognitive abilities that have been found to correlate with math achievement (e.g. Bull & Johnston, 1997; Geary, 1993; Koontz & Berch, 1996), though many of these correlate with vocabulary size which we did control for.

It remains possible that the correlations between ANS acuity and math ability shown here were mediated by a third unknown factor. Third, our work leaves open the possibility that other cognitive factors beyond ANS acuity make significant contributions to children's overall number sense. Indeed the ANS is only one core cognitive ability relevant to success in school mathematics. Future work will be needed to determine how each of these factors relates to success in school.

In summary, we found that the acuity of preschoolers' Approximate Number System correlated significantly with their math ability even when controlling for age and vocabulary size. Our study thereby supports the notion of a tight link between a primitive sense of number and more formal math abilities.

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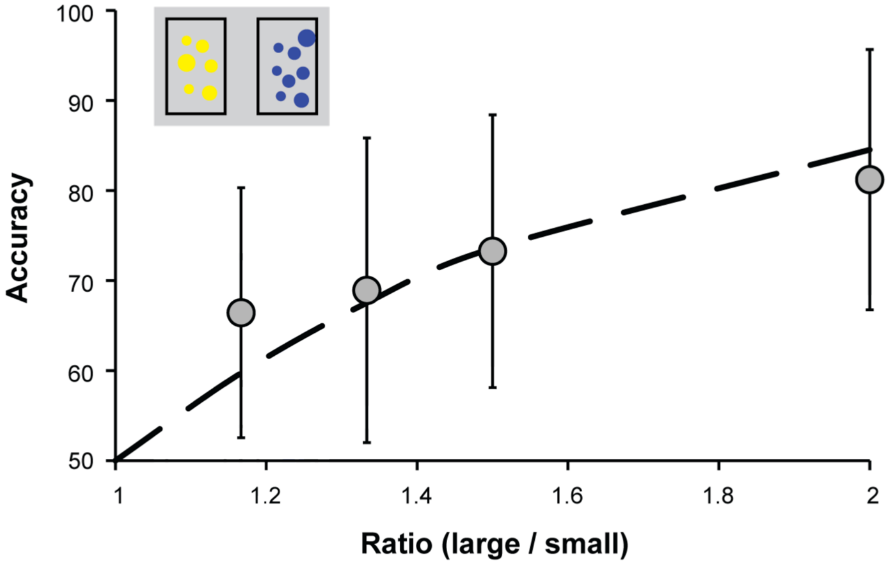
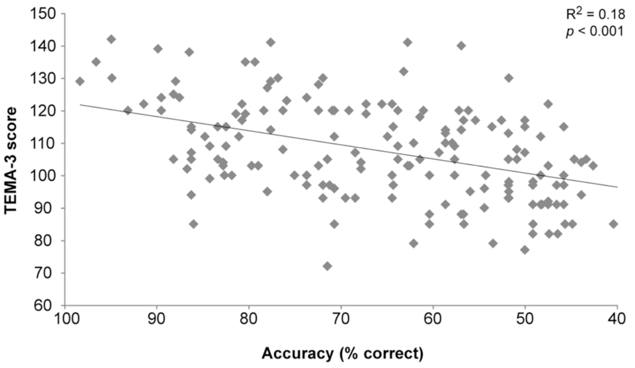


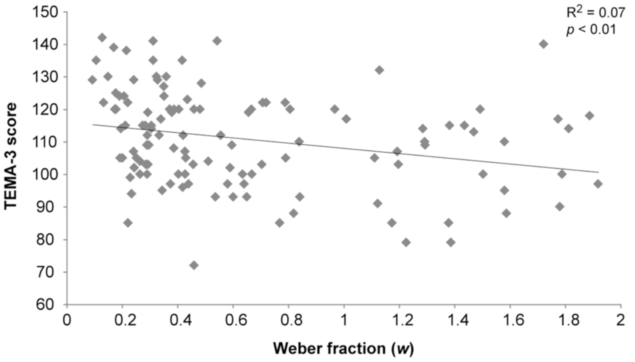
Figure 1.

Insert: Sample stimulus used in ANS acuity task. Participants indicated whether there were more yellow or more blue dots. Accuracy (percent correct) on the ANS acuity task is plotted as a function of the ratio between numerical values. Error bars indicate the individual differences in Weber fraction (*w*) and reflect +/− 1 standard deviation.

A: Accuracy



B: Weber fraction



C: Response time

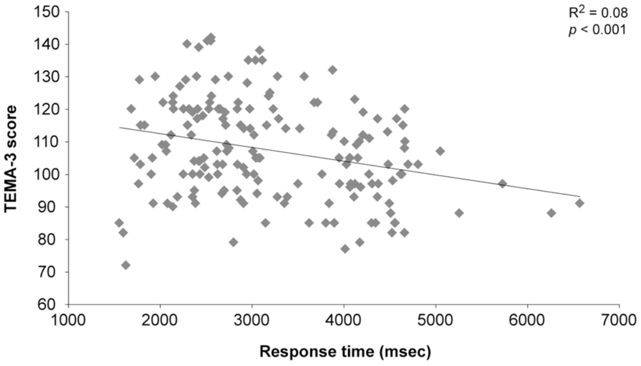


Figure 2.

Correlations between standardized math scores (TEMA-3) and ANS acuity as measured by (A) accuracy, (B) Weber fraction, and (C) response time.

Table 1

Linear regression analyses predicting math ability using vocabulary size (DVAP score), response time (RT), and accuracy or Weber fraction (*w*) on the ANS acuity task (all age controlled) as possible predictors

	Model incl. accuracy and RT			**Model incl. *w* and RT**		
R^2	0.19			0.16		
F-statistics	F(3,156) = 11.30			F(3,110) = 6.83		
p-statistics	$p < 0.001$			$p < 0.001$		
	Predictor	r_p^2	*p*	Predictor	r_p^2	*p*
	DVAP	0.01	0.30	DVAP	0.01	0.20
	RT	0.05	<0.01	RT	0.08	0.001
	Accuracy	0.13	<0.001	*w*	0.06	<0.01