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Developmental Change in the Acuity of Approximate Number and Area Representations

Darko Odic, Melissa E. Libertus, Lisa Feigenson, and Justin Halberda
Johns Hopkins University

From very early in life, humans can approximate the number and surface area of objects in a scene. The ability to discriminate between 2 approximate quantities, whether number or area, critically depends on the ratio between the quantities, with the most difficult ratio that a participant can reliably discriminate known as the Weber fraction. While developmental improvements in the Weber fraction have been demonstrated for number, the developmental trajectory of improvement in area discrimination remains unknown. Here we investigated whether the development of area discrimination parallels that of number discrimination. We tested forty 3- to 6-year-old children and adults in both a number and an area discrimination task in which participants selected the greater of 2 quantities across a range of ratios. We used formal psychophysical models to derive, for each participant and each age group, the Weber fraction for both number and area discrimination. We found that, like number acuity, area acuity steadily improves during childhood. However, we also found area acuity to be consistently higher than number acuity, suggesting a potential difference in the underlying mechanisms that encode and/or represent approximate area and approximate number. We discuss these findings in the context of quantity processing and its development.

Keywords: approximate number system, number, area, quantification, development

How humans represent and compare quantities has been an important question for psychologists for at least the past hundred years. In the field of cognitive development, there has been an ongoing effort to discover how infants and children represent and compare different types of quantity, including number, time, and spatial extent (Carey, 2009; Dehaene, 1997; Lipton & Spelke, 2003; Mix, Huttenlocher, & Levine, 2002; Piaget, 1965). These explorations have attempted to uncover which quantity representations are available early in development, the relationship between representations of different quantity dimensions, and whether representations of quantity change over time and with experience.

A wealth of literature in both comparative and developmental psychology demonstrates that human adults, children, infants, and many nonhuman animals including rhesus macaques (Brannon & Terrace, 2000), untrained cotton-top tamarins (Hauser, Tsao, Garcia, & Spelke, 2003), rats (Meck & Church, 1983), guppies (Piffer, Agrillo, & Hyde, 2012), dolphins (Kilian, Yaman, von Fersen, & Güntürkün, 2003), pigeons (Emmerton, 1998), and chimpanzees (Beran & Rumbaugh, 2001; Tomonaga, 2008) share a system for approximating the number of visual or auditory items in a scene (for review, also see Cantlon, Platt, & Brannon, 2009; Dehaene, 2009; Tomonaga, 2008). Evidence suggests that this sense of number is supported by an internal approximate number system (ANS) that rapidly and automatically produces a primitive sense of number—for example, a sense of approximately how many people are in a crowded room, or how many marbles are in a jar.

A key psychophysical signature of the ANS is that it produces behavior that obeys Weber's law: Observers' ability to discriminate two approximate number representations does not depend on the total number of items or the absolute difference between them, but instead on the ratio between the two quantities (Dehaene, 1992; Feigenson, Dehaene, & Spelke, 2004). Thus, when quickly flashed an array of, for example, 20 blue dots and 10 yellow dots (a ratio of 2.0; see Figure 1), both adults and children are fast and accurate at judging that there are more blue than yellow dots (Halberda & Feigenson, 2008). But when shown a more difficult ratio of, for example, 18 blue dots to 15 yellow dots (a ratio of 1.2; see Figure 1), observers are slower and more error prone.

Recent work has suggested that although the ANS always yields ratio-dependent performance, there are important individual differences in which ratios can be reliably discriminated. Thus, while some people may reliably discriminate a ratio of 1.25 (e.g., 8 vs. 10), others may struggle. This acuity of the internal ANS representations is quantified as the individual's Weber fraction, or *w*, where a higher *w* corresponds to greater uncertainty in the number representations, and thus poorer discrimination performance (Halberda & Feigenson, 2008; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011; Libertus, Odic, &

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Darko Odic, Melissa E. Libertus, Lisa Feigenson, and Justin Halberda, Psychological and Brain Sciences, Johns Hopkins University.

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Correspondence concerning this article should be addressed to Darko Odic, Psychological and Brain Sciences, Johns Hopkins University, Ames Hall, 3400 North Charles Street, Baltimore, MD 21218. E-mail: darko.odic@jhu.edu

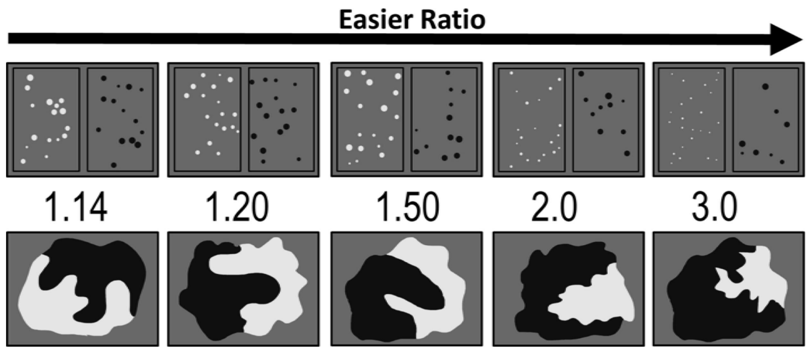

Figure 1. Examples of stimuli used across the five ratios. The top portion depicts the number acuity task ("Who has more dots?") and the bottom depicts the area acuity task ("Who has more goo?").

Halberda, 2012; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). Individual differences in Weber fraction have played an important role in recent theories of number representation. For example, *w* and other measures of ANS functioning correlate with performance on mathematics tasks in preschoolers (Libertus et al., 2011; Mazzocco, Feigenson, & Halberda, 2011), children and adolescents (Gilmore, McCarthy, & Spelke, 2010; Halberda et al., 2008), and adults (Libertus, Odic, & Halberda, 2012; Lyons & Beilock, 2011), suggesting that our approximate number sense may contribute to our ability to perform symbolic mathematics. Additionally, children with dyscalculia, a learning disability specific to mathematics, have been shown to have significantly poorer *w*s than their age-matched peers, further suggesting that the ANS may play a foundational role in number reasoning (Mazzocco et al., 2011; Piazza et al., 2010).

In addition to differing across individuals, Weber fractions change with age. For example, it appears that 6-month-old infants can only discriminate numerical ratios equal to or greater than 2.0 (e.g., 8 vs. 16 dots), whereas 9-month-olds can discriminate arrays as difficult as 1.5 (e.g., 8 vs. 12 dots; Libertus & Brannon, 2010; Xu, Spelke, & Goddard, 2005). As children get older, their ANS acuity continues to improve. Halberda and Feigenson (2008) tested 3-, 4-, 5-, and 6-year-old children on a task in which two collections of images were presented side by side, and children had to indicate which collection was more numerous without counting. Psychophysical modeling of children's judgments revealed a gradual improvement in number acuity over time: At the group level, 3-year-olds exhibited a *w* of 0.53 (3:2 ratio), 4-year-olds of 0.38 (4:3 ratio), 5-year-olds of 0.23 (5:4 ratio), 6-year-olds of 0.18 (7:6 ratio), and adults of 0.11 (10:9 ratio). This developmental trend suggests that numerical estimation abilities continue to change until relatively late in childhood and perhaps well beyond (Halberda, Ly, Wilmer, Naiman, & Germine, 2012).

Decades of psychophysical work on other quantity dimensions have revealed that many of these also obey Weber's law. Discrimination of continuous quantities such as area, time, volume, and line length are also ratio dependent (e.g., Gescheider, 1997; Green & Swets, 1966; Stevens & Guirao, 1963; Teghtsoonian, 1965). And, much like in the case of number, there are individual differences in the Weber fraction for each of these represented dimensions (Stevens & Guirao, 1963).

However, to date, surprisingly little work has directly compared discrimination abilities for different types of quantities. Some evidence suggests that rats discriminate time and number with identical Weber fractions following training (Meck & Church, 1983), and Droit-Volet, Clément, and Fayol (2008) suggested that, under certain conditions, time and number may show similar Weber fractions in 5-year-old children. In human adults, Castelli, Glaser, and Butterworth (2006) found evidence that different regions of the intraparietal cortex were responsible for encoding number versus surface area, but also that observers' average performance accuracy was very similar for number and area judgments. Recently, young 3-year-olds have been shown to learn the meaning of the word *more* in context of both approximating number and approximating area, suggesting an underlying similarity between these two dimensions (Odic, Pietroski, Hunter, Lidz, & Halberda, 2012). Finally, in human infants, 6-month-olds have been shown to discriminate a ratio of 2.0 for the area of an Elmo face (Brannon, Lutz, & Cordes, 2006), and also to discriminate the same 2.0 ratio when comparing the number of elements in an array (Xu & Spelke, 2000), the duration of an auditory event (Brannon, Suanda, & Libertus, 2007; VanMarle & Wynn, 2006), and the speed of an object's motion (Möhring, Libertus, & Bertin, 2012). However, not all quantity dimensions elicit the same acuity in 6-month-olds. Infants fail to dishabituate to a 2.0 ratio change in the cumulative area of an array containing many items, as well as to the size of the individual items in a large array (Cordes & Brannon, 2008, 2011).

Although these previous studies start to suggest a relationship between various quantity representations, much remains to be learned. First, although we know some details concerning the acuity and development of approximate number representations, we know very little about acuity in other domains, including surface area. Second, in humans, the small amount of existing data showing similar acuity for different quantity dimensions has focused primarily on infants (i.e., prior to 12 months) and has not detailed developmental changes in acuity, leaving open the question of whether the acuity of different quantity dimensions increases in parallel across development. Third, existing explorations have largely pooled data across groups and largely ignored the issue of individual differences within and across different quantity dimensions—for example, whether higher acuity in one dimension predicts higher acuity in another.

Here we investigated the developmental trajectory of the acuity of children's representations of number and surface area. Previous work has mapped out developmental changes in acuity for number at the group level for children between 3 and 6 years (Halberda & Feigenson, 2008), and has shown that 6-month-old infants show similar sensitivity to approximate number and the approximate area of a single visual item (Brannon et al., 2006). In the present experiment, we extended this work by examining whether area acuity changes in the same manner as number acuity in children between 3 and 6 years of age. By testing the same participants on both a number and an area acuity task, we also asked whether individual differences in acuity were consistent across these two types of quantity.

In the present experiment we tested 3-, 4-, 5-, and 6-year-old children and adults in both a number and an area acuity task. Participants saw briefly presented displays containing two different colors of dots (number acuity task) or a shape that was partially filled with one color and partially filled with another color (area acuity task). Participants had to indicate which color was more numerous or larger in extent. We varied the ratio between the colors across trials, then used psychophysical modeling of participants' performance across these different ratios to determine the *w* of each participant's number and area representations.

Method

Participants

We tested 40 participants, with eight participants in each age group: 3-year-olds, 4-year-olds, 5-year-olds, 6-year-olds, and adults. The average age within each group was 3.72 years (SE = 0.06; six boys), 4.46 years (SE = 0.11; four boys), 5.64 years (SE = 0.09; three boys), 6.53 years (SE = 0.09; three boys), and 19.25 years (SE = 0.45; four males). Children were recruited by phone and were tested individually at the Johns Hopkins Laboratory for Child Development. After testing, children received a small gift (e.g., t-shirt, book, or stuffed animal) to thank them for their participation. Children were mostly Caucasian and middle class, with parents who had completed some postsecondary education. Adults were Johns Hopkins University undergraduates who volunteered to participate for course credit. Eighteen additional children participated, but were not included in the final sample because of failure to complete both tasks (n = 17) or technical issues during testing (n = 1). Out of the 17 children who failed to complete both tasks, 11 were 3-year-olds, five were 4-year-olds, and one was a 6-year-old; in each case, the child could not sit through both the tasks (i.e., 100 trials total) and became inattentive. Adult participants and parents of child participants gave written informed consent prior to the experiment.

Materials

All participants included in the final sample completed both the number and area acuity tasks. For both tasks, materials consisted of a laptop computer and five Sesame Street character cutouts (Big Bird, the Count, Elmo, Grover, and Oscar) that could be attached to the sides of the laptop screen using Velcro tape. All tasks were administered on a MacBook laptop with a 13-in. screen with custom-made Java programs to display the stimuli.

In the number acuity task, participants saw two spatially nonoverlapping arrays of dots displayed side-by-side, in colors consistent with the characters used (e.g., yellow dots for Big Bird, red dots for Elmo, etc.). Each dot array appeared within a rectangular frame that designated that character's "box" (see Figure 2). Participants were told to indicate which character had more dots. Each array contained between 8 and 24 dots, ranging in size from 0.3 to 1.2 cm in diameter. Arrays instantiated one of five ratios (calculated by dividing the larger number of dots by the smaller): 1.14 (e.g., 16:14 dots), 1.2 (e.g., 18:15), 1.5 (e.g., 18:12), 2.0 (e.g., 20:10), and 3.0 (e.g., 24:8). On half the trials, the more numerous array also had the greater cumulative area (congruent trials; e.g., if there were twice as many blue dots, there was also twice as much blue total area). On the other half of trials, the less numerous array had the greater cumulative area (incongruent trials; e.g., if there were twice as many blue dots, there was twice as much yellow total area). Thus, if participants attempted to use a nonnumerical strategy (e.g., choosing the array with the larger total area), this would be revealed as a difference between congruent and incongruent trials (Barth, 2008; Hurewitz, Gelman, & Schnitzer, 2006; Tokita & Ishiguchi, 2010).

In the area acuity task, participants saw a single irregular shape (described to participants as "goo") that was divided into two colored regions, with color again corresponding to the characters used (see Figure 2). Participants were told to indicate which character "had more goo." The two colored regions within each shape were controlled for total perimeter: On approximately half the trials, the larger area also had the larger perimeter, and on the other half, the smaller area had the larger perimeter, making surface area the only reliable cue for the task. As in the number acuity task, each array instantiated one of five ratios. A custom-made Python program counted the total number of pixels of each

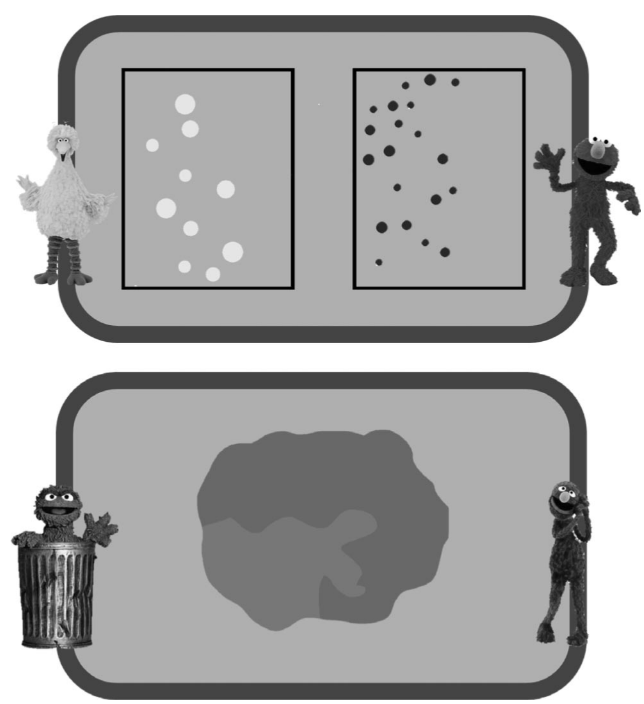

Figure 2. Illustrations of the experimental displays; the top portion depicts the number acuity task and the bottom portion depicts the area acuity task. The ratio in both these images is 2.0. The characters were affixed with Velcro to the side of the monitor (represented by gray outline).

color in each goo image, and ratio was calculated by dividing the total number of pixels in the larger area by that in the smaller area. The ratios used were the same as in the number acuity task, that is, 1.14, 1.2, 1.5, 2.0, and 3.0.

In order to best equate the number and area acuity tasks, both number and area stimuli had separate regions for the dots or colored regions. Thus, dots were physically separated by large rectangles into two regions, while the goo was clearly divided into two colored regions. This allowed factors like eye movements between the blue and yellow dots or regions to be roughly equated (for results in adults with spatially intermixed squares and blobs, see Castelli et al., 2006).

Procedure

All participants were tested individually in a quiet room by a trained experimenter. Participants sat approximately 50 cm from the computer screen. In the case of children, parents sat in a corner of the room, positioned so that they could see their child but could not see the stimuli, thereby preventing any cuing effects. Half of the participants began with the number acuity task, and half with the area acuity task.

Before each task, participants were introduced to the two Sesame Street characters, which were randomly chosen for each participant for each task. Different characters were used for the two tasks to help maintain interest. After their introduction, the character cutouts were attached to the sides of the monitor. Adult participants were tested using this same procedure with Sesame Street characters and were told that the task was created for children and that the instructions would be given using child-friendly language, but that they should try their best to make the necessary discriminations (i.e., not pretend to be a child).

In the number acuity task, participants were shown the empty frames on the left and the right sides of the screen and were told, for example, “Big Bird and Elmo played with some dots. This is Big Bird’s box, and this is Elmo’s box. Big Bird keeps his dots in his box, and Elmo keeps his dots in his box.” Dots were then revealed for each character simultaneously, and participants were asked by the experimenter, “Who has more dots?” Adults pushed one of two keys on the keyboard (*F* for “left side character” or *J* for “right side character”) to indicate their response. Children responded by either saying the character’s name or by pointing, and the experimenter pushed the appropriate key on the laptop to indicate the child’s answer. To maintain their motivation, participants received feedback after each trial in the form of a high tone for a correct answer and a low tone for an incorrect answer. Pilot testing showed that children needed about six practice trials at the beginning of the number acuity task in order to understand the game (see also Halberda & Feigenson, 2008). During these practice trials, the dots for each character appeared first by themselves for 1,500 ms, and then both arrays appeared simultaneously for 1,500 ms. The experimenter provided additional verbal feedback and encouragement throughout the practice. During the test trials, both arrays of dots only appeared simultaneously and remained visible for 1,500 ms, and the experimenter gave only neutral-positive feedback unconnected to the participant’s performance. Participants had an unlimited time window after the dots disappeared in which to respond. They could also respond while the dots were on the screen. Each ratio was presented 10 times, yielding a total of 50 number acuity trials.

In the area acuity task, participants first were introduced to the characters and then began with a practice trial depicting one goo shape that stayed on the screen for 10 s. During this time, the experimenter told the participant that, for example, “Oscar and Grover played with some goo.” The experimenter then dragged his/her finger around each region of the goo and said, for example, “This is the Oscar’s goo, and this is Grover’s goo.” Participants were then asked, “Who has more goo?” Pilot testing showed that children did not require more than one practice trial to understand this task and that they became fatigued during further practice trials, and therefore the test trials began immediately after this single practice trial. During the test trials, the goo remained visible for 1,500 ms. Adults pushed a button (*F* or *J*) to indicate their response. Children responded by either saying the character’s name or by pointing, and the experimenter pushed a button to indicate the child’s answer. As with the number acuity task, each trial was followed by computerized feedback, and each ratio was presented 10 times, yielding a total of 50 area acuity trials.

Results

We first analyzed the data in terms of participants’ total percent correct (see Table 1). A 5 (age group) × 2 (task order) × 2 (task) × 5 (ratio) mixed measures analysis of variance (ANOVA) yielded significant main effects of age, $F(4, 30) = 13.41$, $p < .001$, $\eta_p^2 = .64$; task, $F(1, 30) = 26.98$, $p < .001$, $\eta_p^2 = .47$; and ratio, $F(4, 120) = 89.96$, $p < .001$, $\eta_p^2 = .75$, and no main effect of task order, $F(1, 30) = 1.32$, $p = .26$, $\eta_p^2 = .04$, nor any significant interactions with task order (all *p*s > .20). Therefore, task order was dropped as a factor from subsequent analyses. In addition, we found a significant Task × Age interaction, $F(4, 30) = 2.90$, $p < .05$, $\eta_p^2 = .28$, and a significant Task × Ratio interaction, *F*(4,

Table 1
Percent Correct and Estimated Weber Fractions for Each Age Group for the Number and Area Tasks

	Number acuity task				Area acuity task			
Age group	Percent correct (*SEM*)	*w* (*SEM*)	r^2	Nearest whole number fraction	Percent correct (*SEM*)	*w* (*SEM*)	r^2	Nearest whole number fraction
3-year-olds	71.50 (2.16)	0.527 (0.07)	0.89	3:2	76.25 (2.46)	0.442 (0.07)	0.99	3:2
4-year-olds	74.75 (3.06)	0.461 (0.09)	0.81	3:2	86.25 (2.89)	0.296 (0.14)	0.88	4:3
5-year-olds	80.25 (2.40)	0.307 (0.04)	0.99	4:3	87.00 (1.96)	0.190 (0.04)	0.99	6:5
6-year-olds	85.00 (1.25)	0.227 (0.03)	0.99	5:4	90.50 (2.13)	0.148 (0.03)	0.86	7:6
Adults	91.75 (1.16)	0.132 (0.02)	0.99	9:8	93.00 (0.65)	0.115 (0.01)	0.89	10:9

120) = 3.18, $p < .05$, $\eta_p^2 = .10$. As shown in Figure 3 and Table 1, the significant ratio effect reflected that all age groups showed ratio-dependent performance consistent with Weber's law for both the number and area acuity tasks. The significant task effect arose as participants performed better overall on the area acuity task than the number acuity task, although a significant Task × Age linear contrast suggests that this difference may diminish with age, $F(4, 30) = 2.9$, $p < .05$. The significant age effect reflects that performance improved with age in both number and area acuity tasks. Finally, the significant Task × Ratio interaction shows that performance in the area acuity task reached asymptote more quickly than performance in the number acuity task.

To investigate the effects of nonnumerical dimensions on number judgments, we examined performance on trials in which the two quantity dimensions were congruent versus incongruent. Previous work by Hurewitz et al. (2006) and Tokita and Ishiguchi (2010) found that adult observers sometimes use the total area of dots as the basis for ordinal judgments rather than their numerosity, and therefore perform better on congruent trials (where the more numerous array has more surface area) than incongruent trials (where the less numerous array has more surface area; but see Barth, 2008). We performed a 2 (trial type: congruent or incongruent) × 5 (age) mixed measures ANOVA that yielded no effect of trial type, $F(1, 35) < 1$, $\eta_p^2 = .01$, and no interaction with age, $F(4, 35) = 1.547$, $p = .21$, $\eta_p^2 = .15$. Thus, it appears that participants in our number acuity task were successful at ignoring area, and that participants' reliance on number over area in this task did not change with development.

Given that participants' behavioral performance obeyed Weber's law, we next examined each individual participant's performance in order to determine their Weber fraction (w) for each task. To determine each participant's w, we applied a commonly used psychophysical model previously used by Green and Swets (1966), Halberda and Feigenson (2008), Libertus et al. (2011), Piazza et al. (2010), and Pica, Lemer, Izard, and Dehaene (2004):

$$percent\ correct = \frac{1}{2} erfc\left(\frac{n_1 - n_2}{\sqrt{2}w\sqrt{n_1^2 + n_1^2}}\right) * 100$$

The model assumes that the two underlying representations of approximate number or approximate area generated on each trial are distributed along a continuum of Gaussian random variables (with one having mean of n_1, and the other with a mean of n_2). An important implication of this model is that the two different numbers or areas on each trial will often have overlapping representations. As the two quantities become more similar (i.e., approach

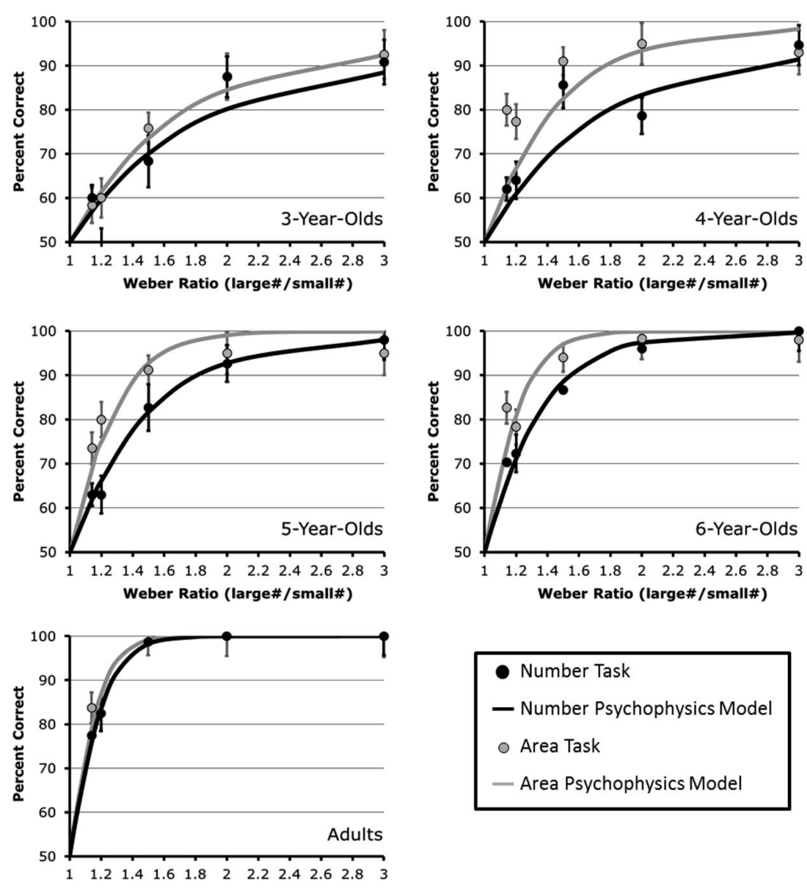

Figure 3. Percent correct (± *SEM*) on the number acuity task (black) and the area acuity task (gray) averaged for each presented ratio. Fits from the psychophysical model are overlayed.

a ratio of 1.0), their representations will increasingly overlap and participants should have increasing difficulty determining which quantity is larger. The model uses the complementary error function *erfc* to determine the expected percent correct at each possible ratio, producing a smooth function that can be compared to the actual observed data.

This model has only a single free parameter—the Weber fraction (w)—which indicates the amount of noise in the underlying Gaussian representations (i.e., the standard deviation of the n_1 and n_2 Gaussian representations where $SD_{n1} = w \times n_1$). Larger w values indicate higher representational noise and, thus, poorer discrimination across ratios (lower Weber fractions indicate better performance). For each participant, the w value that minimized the least squared error (i.e., the squared difference between predicted and actual data) was selected as the best fitting one; this procedure was also used by Halberda and Feigenson (2008).

We applied the model to each participant individually for both the number and area acuity tasks. This analysis revealed that one 4-year-old child was 2.7 standard deviations from the mean in both number and area w values. Removing this participant's data did not change the conclusions of any of the analyses, and therefore the statistics reported here include all available data. We first examined the development of number and area acuity by comparing w scores averaged across age groups (i.e., w was computed using the combined data of all participants within each age group). The average w for each age group in the two tasks is presented in Table 1. In general, r^2 values, which measure the fit between the model and the data, were very high (see Table 1), and the w values obtained here agree with w values previously reported for adults' and children's numerical discriminations (Halberda & Feigenson, 2008; Libertus et al., 2011; Piazza et al., 2010). Next, we examined differences in w scores between the two tasks through a 5 (age) × 2 (task) mixed measures ANOVA, which revealed both a main effect of age, $F(4, 35) = 6.22$, $p < .01$, $\eta_p^2 = .42$, with w decreasing with age, and a main effect of task, $F(1, 35) = 10.25$, $p < .01$, $\eta_p^2 = .23$, with w being lower in the area acuity task than the number acuity task, but no Task × Age interaction, $F(4, 35) < 1$, $\eta_p^2 = .08$. This again confirms that quantity representations of both number and area improve over development, and that area is consistently better. The absence of an interaction between these factors suggests that the improvement over time is equivalent in the two domains.

Recall that auditory feedback (i.e., low or high tone via the computer) was given to participants after each trial as a function of their accuracy. To see whether this feedback drove participants' performance, we divided each task into two halves, and, for each, computed the best fitting w and the average percent correct. If feedback significantly affected performance, we should find that performance during the second half of testing is better (i.e., shows higher accuracy and lower w) compared to the first. A 5 (age) × 2 (task) × 2 (half: first, second) mixed measures ANOVA did not reveal a main effect of half in either the average percent correct,[1] $F(1, 35) = 0.95$, $p = .34$, $\eta_p^2 = .03$, or w scores, $F(1, 30) = 1.01$, $p = .42$, $\eta_p^2 = .03$. There were no significant interactions between half, age, and/or task (all ps $> .35$). This suggests that feedback had no major impact that changed over the course of the experiments and that there was no developmental change in the use/nonuse of feedback.

Next we addressed the issue of developmental change in number and area representations. We extrapolated the developmental trajectory of w values from infancy through adulthood for both area and number discrimination (see Figure 4), using estimates from previous work with 6-month-old infants (Brannon et al., 2006; Lipton & Spelke, 2003; Xu & Spelke, 2000). The fit was obtained using a least squares method over the grouped data for each age group. Consistent with previous work (Halberda & Feigenson, 2008; Piazza et al., 2010), we fit a power function to both area and number acuity (see Figure 4). The power function is described by two parameters—the constant and the exponent; smaller values in both of these parameters indicate faster growth of w over time. In the case of number, we found that the best fitting power function ($r^2 = 0.91$) had a constant of 0.78 and an exponent of -0.57, consistent with previous work (Halberda & Feigenson, 2008; Piazza et al., 2010). In the case of area, we found that the best fitting power function ($r^2 = 0.91$) had a constant of 0.65 and an exponent of -0.64, with both parameter values suggesting a more rapid improvement in area acuity than number acuity across time.

One concern regarding this analysis may be that the estimates of number and area acuity provided by infants are less reliable, given the small numbers of trials given to infants and the difference in testing procedure (habituation for infants vs. ordinal comparison for children and adults). To address this concern, we refit our data, excluding the infant data. In the case of number, we found that the best fitting power function ($r^2 = 0.91$) had a constant of 1.124 and an exponent of -0.76. In the case of area, we found that the best fitting power function ($r^2 = 0.84$) had a constant of 0.95 and an exponent of -0.84. Though the exponent values change once infant estimates are excluded, the fits even more robustly suggest a faster growth of area acuity compared to number acuity.

Next we examined w scores on an individual level. We found a strong negative correlation between age (in days) and w for both number, $r(39) = -0.54$, $p < .01$, and area acuity, $r(39) = -0.34$, $p < .05$—a replication of our previous analysis that number and area acuity increased with age when examined at the group level. Furthermore, these correlations remained significant even when adult subjects were removed, number: $r(31) = -0.63$, $p < .01$; area: $r(31) = -0.54$, $p < .01$, suggesting that the increase in acuity over age is not due to the large jump between 6 years and adulthood.

Before asking whether w scores are correlated between number and area, it is important to control for these large developmental improvements in w across ages, and for the differences in variance across age groups (i.e., if one simply performs a linear regression on the raw w scores for number and area, one will find a correlation simply because younger children are higher on both number and area). Our sample allowed us to determine whether each participant had a better or a worse w relative to his or her age group by controlling for age trends. For both area and number acuity, we created age-dependent z scores for each child's w that indicated

[1] This analysis was performed on w scores and on average percent correct because, for five of the participants (one 3-year-old, one 5-year-old, and three 6-year-olds), the model could not find a best fitting w for one of the two halves (in either the number or the area acuity task). This was in part due the smaller number of data points. Using average percent correct allowed us to use everyone's data.

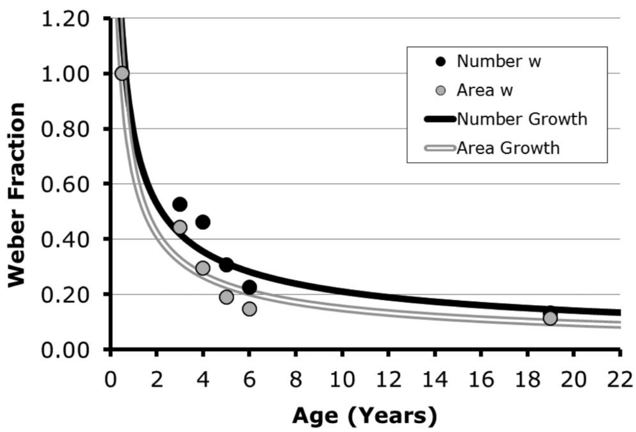

Figure 4. Estimated change in w as a function of age. The 6-month point is taken from Lipton and Spelke (2003), Xu and Spelke (2000), and Brannon et al. (2006) and the remaining points are from the present study.

how well or poorly the child performed relative to his or her age group (i.e., number of standard deviations above or below the age group mean). If individuals with better number w scores also tend to have better area w scores relative to their peers, then a graph of number z score and area z score should show a clear linear trend. As can be seen in Figure 5, we did not find such a trend. A linear regression of number w z score and area w z score was not significant, $r(39) = 0.11$, $p = .42$. Thus, once age trends and variability in the age groups were controlled for by standardizing participants' scores, number and area acuity did not appear to correlate. This is a noteworthy result and is consistent with number and area relying on similar (e.g., both obeying Weber's law) but distinct systems (Castelli et al., 2006).

We investigated this lack of a correlation further by controlling for any unreliability of the number and area acuity measures, that is, the maximum possible correlation between any two measures is limited by the internal consistency of each (Schmidt & Hunter, 1996; Wilmer, 2008). The statistical solution for this problem—attenuation correction—recomputes the correlation between two measures as a ratio of the calculated correlation to the maximum possible correlation given the internal consistency of the two tasks. We performed a random split half of the two tasks and calculated Cronbach's alpha (a measure of internal consistency) of .54 for the number acuity task and .74 for the area acuity task. Once corrected, the correlation between the standardized area and number acuity scores was still not significant, $r(34) = 0.18$, $p = .21$, further suggesting that area acuity and number acuity rely on two distinct systems.

Discussion

In this experiment, we investigated the development and relationship between two types of quantity representations: approximate number and approximate area. Our results revealed three key findings.

First, we found that both area and number discrimination obeyed Weber's law. This expands on a wealth of previous research showing ratio-dependent performance of number discrimination across the life span, and suggests that computing the approximate area contained within a complex, irregular shape also exhibits this type of ratio dependence—a result that has remained largely unexplored in a literature that has focused on the representation of area for geometric shapes (Anderson & Cuneo, 1978; Morgan, 2005; Nachmias, 2008). This similarity between number and area performance is consistent with the possibility that the underlying representational format is similar across these two types of quantity (see also Cantlon et al., 2009; Feigenson, 2007).

Our second conclusion is that area representations appear to have higher acuity than number representations. This is surprising, given that at 6 months of age, infants have been suggested to have identical discrimination thresholds for these two quantities (Brannon et al., 2006). It is important to note, however, that it has not been possible to definitively determine a Weber fraction for infants as they do not provide multiple decision trials across ratios. The rough estimate of their underlying Weber fraction discussed in the literature has been abstracted by observing infants' tendency to dishabituate to a change between larger but not smaller ratios. Such data cannot determine a Weber fraction with specificity (which is more appropriately understood as an internal scaling factor or estimate of internal noise; e.g., Laming, 1986). For this reason, infants might also have distinct Weber fractions for number and area that methods like habituation are unable to detect.

Third, we found that number and area acuity followed a similar growth function across development, but with improvements in area acuity occurring more quickly than improvements in number acuity. This suggests both an underlying similarity and an important difference in the cognitive representation of these two quantities. One possibility is that purely maturational factors, such as improvements in working memory, inhibition, task switching, and so on, lead to the sharpening of quantity representations over time, and that these same maturational factors lead to a faster growth in area acuity. Another possibility is that experience with manipulating number (e.g., counting or learning formal mathematics) changes the acuity of number discriminations (Verguts & Fias, 2004) in a manner distinct from the processes affecting area discrimination. Although our results cannot directly contribute to this debate, given that the period between 3 and 6 years is one during which counting and mathematics skills undergo rapid de-

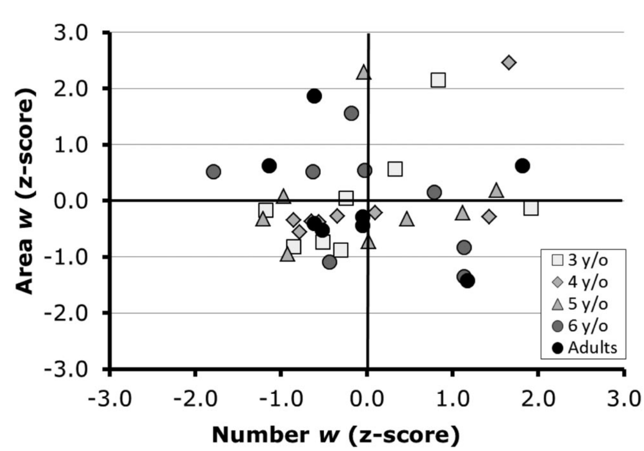

Figure 5. A scatterplot showing the relationship between the w values z-scored to each individual age group's mean and standard deviation. There is no relationship between number and area w at any age. y/o = years old.

velopment (Carey, 2009; Gelman & Gallistel, 1978), the faster growth of area over number acuity may imply an important role for maturational factors. A similar conclusion in favor of maturational factors was made by Piazza and colleagues (2010), who argued that the most rapid period of number acuity change—between infancy and childhood—is one during which there is less direct experience with number manipulation than the period between childhood and adulthood, where the growth of number acuity is slower. However, the relative roles of maturation and experience warrant much future investigation.

In previous theorizing, the finding of similar discrimination abilities for area and number has been used as evidence of a single mechanism for representing diverse dimensions of quantity, for example, area, number, time, and length (Brannon et al., 2006; de Hevia & Spelke, 2009; Lourenco & Longo, 2010; Meck & Church, 1983; Walsh, 2003). This work has almost exclusively focused on infants and nonhuman animals, and has used both a similar *w* between different dimensions and transfer from one dimension onto another as evidence for a single magnitude system. However, the difference in acuity observed in our number and area acuity tasks suggests that these representations may in fact be processed differently throughout childhood and adulthood. It may be that the format of different quantity representations is similar, but that the content is not. Both number and area representations could be represented as noisy distributions with scalar variability, but each may be encoded, represented, and computed over separately (see also Castelli et al., 2006). This idea is supported by an absence of a significant correlation between number and area acuity in our sample once age is controlled.

One possibility, suggested previously by Lourenco and Longo (2010; see also Piaget, 1965), is that, in infancy, the quantity dimensions belong to a common system but that, over time, this system diverges into several dedicated systems for, for example, number and area approximation. Although nothing in our data stands against this possibility, it remains unclear how a single mechanism could computationally extract both number and area information from the visual stimulus. One important future direction will be in extending this kind of work into infancy, neural systems, and computational modeling (e.g., attempting to find whether there are correlations between number and area acuity prior to 3 years of age).

Although many authors have argued in favor of the single quantity mechanism hypothesis, our finding that number and area representations are distinct accords with some previous findings. For example, Castelli and colleagues (2006) found a dissociation between the regions of the intraparietal sulcus (IPS) that were active during adults' processing of area versus number, and suggested that a dedicated portion of IPS is used for encoding descriptors of discrete stimuli (e.g., number). Pinel, Piazza, Le Bihan, and Dehaene (2004) also found a dissociation in adult IPS between processing of number and size information, although in their case, the numerical stimuli were Arabic digits. One highlight of Castelli and colleagues' work was their effort to use very similar stimuli to test observers' representations of number and area. However, despite the neural dissociation, Castelli and colleagues failed to find a significant difference between behavioral performance on number and area tasks, and they did not determine the underlying Weber fraction for either dimension. If we analyze participants' average percent correct scores in our task, we also do not find a difference between adults' performance in number and area, suggesting that *w* may be a more sensitive measure. Combined, we view this work with adults as largely converging with the work we report here, both suggesting that although number and area are processed in brain regions showing similar information-processing characteristics, there are dissociations between number and area in the content of the representations and perhaps in the procedures that encode the relevant visual dimensions (see also Barth, 2008; Cantlon et al., 2009; Hurewitz et al., 2006; Tokita & Ishiguchi, 2010).

What role might approximate number and area representations play throughout the life span? As mentioned earlier, several recent studies have identified a correlation between individual differences in *w* scores (measured in numerical discrimination tasks such as that used here) and symbolic mathematical abilities (Gilmore et al., 2010; Halberda et al., 2008; Halberda et al., 2012; Libertus et al., 2011; Lyons & Beilock, 2011). These results are observable as early as preschool, and appear to be predictive of later mathematical abilities (Mazzocco et al., 2011). Here we found that children's and adults' area discriminations, much like their number discriminations, display large individual differences both within and between age groups. This raises the question of whether individual differences in the *w* of area representations might also correlate with or predict later cognitive abilities. For example, area acuity might correlate with spatial or geometrical reasoning abilities, or with mathematical ability. Future work might explore this relationship and the possibilities for engaging these core systems to improve children's reasoning about space and quantity.

Another open question concerns the underlying encoding mechanisms for area and number approximation. Work in both monkey electrophysiology and computational modeling has provided several clues as to how number may be encoded and represented by neuronal systems (Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Dehaene & Changeux, 1993; Nieder, 2005; Stoianov & Zorzi, 2012; Verguts & Fias, 2004). However, little research has explored the issues of the neural encoding of area. Work by Gestalt psychologists and psychophysicists suggested that area estimation is highly affected by stimulus shape (Gigerenzer & Richter, 1990; Morgan, 2005; Nachmias, 2008). One possibility is that area is extracted through a mechanism that operates over surface contours of objects and calculates their dimensions. This research has suggested that the area of geometric objects, such as rectangles and ellipses, is estimated through their aspect ratio (height:width) and/or by estimating their diameter (Anderson & Weiss, 1971; Chong & Treisman, 2003; Morgan, 2005). Such a simple mechanism, however, could not account for the irregular stimuli presented here (see also Teghtsoonian, 1965). An alternative possibility is that area is extracted from the combined response of low spatial-frequency detectors (Dakin et al., 2011). Thus, future work should address the question of how the visual system computes estimates of area across various figure shapes.

Our work demonstrates both important similarities and differences in the abilities of young children and adults to represent approximate number and area. We found that although both abilities obey Weber's law and demonstrate a similar growth pattern, by at least 3 years of age children's area representations are more precise than, and do not correlate with, the acuity of their number representations. Future work will be needed to characterize the ways in which representations of discrete and continuous quantity may interact across the life span.

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